Ma 635. Real Analysis I. Lecture Notes

VIII. MEASURE

8.1 **Definition.** Lebesgue measure μ of interval (a, b) of numerical line is $\mu_L(a, b) = b - a$.

8.2 **Definition.** Lebesgue outer measure of a set E is

$$\mu_L^*(E) = \inf \left\{ \sum_{n=1}^{\infty} \mu_L(I_n) : \ E \subset \bigcup_{n=1}^{\infty} I_n \right\}$$

where the infimum is taken over all coverings of E by countable unions of intervals.

8.3

$$\begin{aligned}
&\mu_L^*(\emptyset) = 0 \\
&\mu_L^*(\text{countable set}) = 0 \\
&\mu_L^*(a, b) = \mu_L(a, b) \\
&\mu_L^*(E) \le \mu_L^*(F) \quad \text{if } E \subset F \quad \text{(monotonicity)}
\end{aligned}$$

8.4 **Definition** If $\{(a_i, b_i)\}$ is a collection of pairwise disjoint open intervals then

$$\mu_L \left(\bigcup_{i=1}^{\infty} (a_i, b_i) \right) := \sum_{i=1}^{\infty} (b_i - a_i).$$

The sum may be infinite.